Comparative Analysis of Solid and Hollow Shafts Using Modified Goodman Method

Musa M. Allafi [1*], Osama M. AlMakhzom [2]
[1] Higher Institution of Marine Science Technologies – AL khoms, Libya
[2] Engineering and Information Research Center. Bani Walid, Libya
***Corresponding author: mm.allafi@gmail.com**

Abstract:

Power shafts are the most crucial components of a machine, and they are frequently subject to fatigue due to variable external loads. The goal of this research is to design solid and hollow shafts with constant load and torque in order to anticipate the percentage reduction in shaft mass and change in diameter. Modified Goodman approach was used to calculate the shaft's diameter, which was then compared to the maximum shear stress method. The results show that hollow shafts have lower mass than solid shafts with the same material and strength, while hollow shafts have higher outer diameter. When using maximum share stress theory, the results show that the diameter of the designed shaft increases slightly compared to Goodman method.

Keywords: Power Shaft, Hollow Shaft, Solid Shaft, Mod_Goodman Method, Fatigue.

مقارنة تحليلية للأعمدة الصلبة والمجوفة باستخدام طريقة جودمان المعدلة

موسى محمد اللافي [1*]، اسامة مفتاح المخزوم [2]
[1] المعهد العالي لتقنيات علوم البحار – الخمس، ليبيا
[2] مركز البحوث الهندسية وتقنية المعلومات – بني وليد، ليبيا

الملخص:

أعمدة القدرة هي أهم مكونات الآلة، وكثيرًا ما تتعرض للكلل بسبب الأحمال الخارجية المتغيرة. يهدف هذا البحث إلى تصميم أعمدة صلبة ومجوفة بحمل وعزم دوران ثابتين لتوقع نسبة الانخفاض في كتلة العمود وتغير قطره. استُخدمت طريقة جودمان المعدلة لحساب قطر العمود، ثم قورنت بطريقة إجهاد القص الأقصى. أظهرت النتائج أن كتلة الأعمدة المجوفة أقل من كتلة الأعمدة الصلبة المصنوعة من نفس المادة والقوة، بينما قطرها الخارجي أعلى. عند استخدام نظرية إجهاد القص الأقصى ، أظهرت النتائج أن قطر العمود المُصمم يزداد قليلاً مقارنة بطريقة جودمان.

الكلمات المفتاحية: أعمدة القدرة ، عمود مجوف، عمود صلب، طريقة جودمان المعدلة ، الكلل.

I. INTRODUCTION

The rotating components that transmit power in machines are called shafts. Typically, they are made of low- or medium-carbon steel and have a circular cross section. They can be solid or hollow. As a ubiquitous and significant rotating machine component, shafts support a variety of transmission components, including gears and pulleys. The main difference between a solid shaft and a hollow shaft is that the cross-sectional area of the hollow shaft is spread away from the axis of rotation, while the cross-sectional area of the solid shaft is spread closer to the axis of rotation. The hollow shaft has a greater moment of inertia than the solid shaft for the same weight per unit length of shaft. Therefore, hollow shafts are stiffer than solid shafts in terms of torsional and bending forces; but for the same objective, a hollow shaft's diameter is larger than a solid shaft's, requiring more space. Due to the advantages of hollow columns, especially their light weight and hardness compared to solid ones, and due to their exposure (whether solid or hollow) to different loads during their work, which generates different stresses on the material of the shafts, it is important to study the effect of these forces and the extent to which the design (outer diameter of the shaft) differs between the hollow and solid types. The Mod-Goodman method is widely used to study and analyze shaft stresses to compute the fatigue effect on the component, as all machines and structures are subject to fatigue (Osgood, 1982 [1]; Stephen, 2018 [2]). The design and optimization of shafts under fatigue loading were investigated by Gujar (2013) [3] and Deepan (2011) [4]. According to Rakesh et al. (2018), aluminum with a BLF composite hollow pit is accurate in terms of weight curtailment, but only by 1.16% when compared to aluminum fineness with a BLF compounded shaft (Rakesh and others, 2018–[5]). In this work, Mod-Goodman method was used to design and analyze stresses of solid and hollow power shafts to verify mass reduction of the designed shaft. Besides that, the increasing of outer diameter and fatigue resistance were considered.

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II. METHODOLOGY

The suggested shaft is 300 mm long, with different diameters contingent on the outcome of the design. As shown in figures 1 and 2, it is subjected to a torque of 300 N.m. and a single force of 1000 N operating on the pulley. Table 1 displays the characteristics of the various carbon steel grades employed in the design. The mod-Goodman equation and maximum shear stress method are used in this work to compute the diameters of the solid and hollow shafts in accordance with the mentioned boundary conditions. The mass of both kinds of shafts is then determined; this process was carried out seven times in accordance with various material grades. Plotting the findings against mod-Goodman lines thus guarantees the finite life of the designed shafts.

Table 1: Material properties.

AISI / CD	S_{ut} (MPa)	S_y (MPa)	ρ (g/cm3)
AISI 1010	370	300	7.87
AISI 1018	440	370	7.87
AISI 1020	470	390	7.87
AISI 1030	520	440	7.85
AISI 1035	550	460	7.85
AISI 1040	590	490	7.84
AISI 1050	620	580	7.85

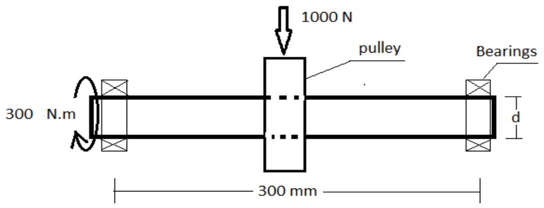

Figure 1: Studied Model (Solid Shaft)

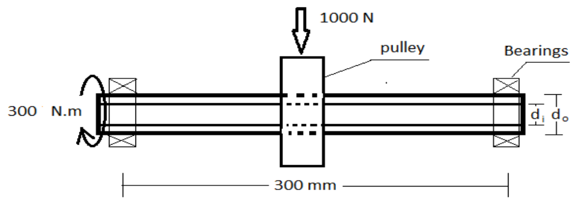

Figure 2: Studied Model (Hollow Shaft)

III. SOLID AND HOLLOW SHAFT DESIGN USING MOD-GOODMAN

The Mod-Goodman equation (1) is used in a tensile region where compression stress has little effect on fatigue safety factor (Stephen K.2018 - [2]). Since the shafts in our case are subjected to bending and torsion, in fluctuating assuming no axial force on the shafts, thus, the bending and torsional stresses have alternating and midrange components, using von Mises stresses formulas (equations 2 and 3) for the two stress elements with fatigue factors (K_f for moment and K_{fs} for shear) and S_{ut} is the ultimate tensile strength, equations 4 and 5 can be obtained (Shigley.2015 - [6]).

$$\frac{1}{n} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

$$\sigma_a' = [\sigma_a^2 + 3\,\tau_a^2\,]^{1/2} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

$$\sigma_m' = [\sigma_m^2 + 3\,\tau_m^2\,]^{1/2} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

For the solid shaft:

$$\sigma_a' = \left[\left(\frac{32K_F M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2} \quad \ldots\ldots\ldots (4)$$

$$\sigma_m' = \left[\left(\frac{32K_F M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2} \quad \ldots\ldots\ldots (5)$$

And for hollow shafts:

$$\sigma_a' = \left[\left(\frac{32K_F M_a}{\pi d_o^3(1-k^4)}\right)^2 + 3\left(\frac{16K_{fs} T_a}{\pi d_o^3(1-k^4)}\right)^2\right]^{1/2} \quad \ldots (6)$$

$$\sigma_m' = \left[\left(\frac{32K_F M_m}{\pi d_o^3(1-k^4)}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d_o^3(1-k^4)}\right)^2\right]^{1/2} \quad \ldots (7)$$

Where, d_o is the outer diameter of hollow shaft and k is d_i/d_o, d_i is the inner diameter of hollow shaft.

Applying these equations to the Mod-Goodman equation (1), taking into consideration rotational shafts such as ours with constant bending and torsion: Mm =Ta=0, the diameter of a specified shaft can be computed using equations 8 and 9 for solid and hollow shafts.

$$d = \left\{\frac{16n}{\pi}\left[\frac{2K_F M_a}{S_e} + \frac{\left(3\left(K_{fs} T_m\right)^2\right)^{1/2}}{S_{ut}}\right]\right\}^{1/3} \quad \ldots (8)$$

$$d_o = \left\{\frac{16n}{\pi(1-k^4)}\left[\frac{2K_F M_a}{S_e} + \frac{\left(3\left(K_{fs} T_m\right)^2\right)^{1/2}}{S_{ut}}\right]\right\}^{1/3} \quad \ldots (9)$$

The calculation parameters that are used in this work are illustrated in table 2 and the endurance limit Se was calculated using Marin equation as below (Shigley.2011 - [6]):

$$S_e' = k_a k_b k_c k_d k_e k_f f_e' \quad \ldots\ldots\ldots\ldots\ldots (10)$$

Using $f_e' = \frac{S_{ut}}{2}$ to predict rotary beam test specimen endurance limit and the results of the equation 10 are illustrated in table 3.

Table 2. calculation parameters

parameter	value
Fatigue safety factor **n**	2.5
fatigue factor for moment K_f	2.14
fatigue factor for shear K_{fs}	3
K (d_i/d_o)	0.6

Table 3: Endurance limits .

AISI / CD	**S_{ut} (MPa)**	Ka Surface Factor (Machined CD)	K_b Size Factor	K_c Loading Factor (Bending)	K_d Temperature Factor (50 C^o)	K_e Reliability Factor (90%)	K_f Miscellaneous-Effects Factor	S'_e
AISI 1010	370	0.941	1	1	1.012	0.897	1	**158.1**
AISI 1018	440	0.899	1	1	1.012	0.897	1	**179.6**
AISI 1020	470	0.883	1	1	1.012	0.897	1	**188.5**
AISI 1030	520	0.860	1	1	1.012	0.897	1	**203.0**
AISI 1035	550	0.847	1	1	1.012	0.897	1	**211.6**
AISI 1040	590	0.832	1	1	1.012	0.897	1	**222.8**
AISI 1050	620	0.821	1	1	1.012	0.897	1	**231.0**

IV. SOLID AND HOLLOW SHAFT DESIGN USING MSS

The diameter of the shaft can be calculated in another way using maximum shear stress method (equation 11) (Steven R. Schmid.2014 - [7]), which does not account for the endurance limit and the fatigue factor of safety. Naturally, this approach forecasts a broad design approximation unless fatigue considerations play a significant role in machine design. This technique was used in this work to determine the diameter of both solid and hollow designed shafts in order to illustrate how fatigue considerations affect machine design. Equations 12 and 13 can be used to determine the diameters of designed shafts.

$$\frac{\pi}{16}\tau_{max}d^3 = [M^2 + T^2]^{1/2} \quad \dots\dots\dots(11)$$

$$d = \left[\frac{16\left[(k_f M)^2 + (k_{fs}T)^2\right]^{1/2}}{\pi\tau_{max}}\right]^{1/3} \quad \dots.\dots(12)$$

$$d_o = \left[\frac{16\left[(k_f M)^2 + (k_{fs}T)^2\right]^{1/2}}{\pi\tau_{max}(1-k^4)}\right]^{1/3} \quad \dots\dots.(13)$$

Where $\tau_{max} = \frac{S_y}{2*n}$

V. VON MISES STRESS COPONENTS

Von Mises stress components (σ'_a and σ'_m) must be identified in order to apply Langer equation (14) (Shigley.2015 - [6]) to study the first-cycle yielding and to graphically employ the Mod-Goodman line. Equations (4 and 5) can be used to compute von Mises stress components for designed diameters while taking Mm=Ta=0 into account.

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} \ldots\ldots(14)$$

VI. RESULT AND DISCUSSION

Results of the Mod-Goodman method has been filled in the table 4. These data were presented by figure 3 and figure 4. The results reveal a significant reduction in mass for hollow shafts compared to solid shafts of the same material and strength. Hollow shafts, on the other hand, have a larger outer diameter than solid shafts; the average mass decrease is 28.7%, while the average diameter increase is approximately 4.6%. According to A. Fierek (2020) [8], the percentage difference between solid and hollow shafts is 39% for a 32 mm diameter. Results of the MSS method was illustrated in the table 5, and presented by figure 5 and figure 6. These findings also reveal a significant decrease in the mass of the hollow shafts while increasing their outer diameter. The average mass reduction is 29.5%, while the average diameter increase is 4%. The outside diameters computed using this method are clearly smaller than those obtained using the Mod-Goodman method. This, of course, is due to the lack of fatigue consideration.

Table 4. Results of Mod-Goodman method

	Solid	Hollow		Solid	Hollow
AISI / CD	d (mm)	do (mm)	di (mm)	mass (kg)	
AISI 1010	44	46	27	3.59	2.57
AISI 1018	41	43	25	3.12	2.27
AISI 1020	40	42	25	2.97	2.11
AISI 1030	39	41	24	2.82	2.05
AISI 1035	39	40	24	2.82	1.9
AISI 1040	38	40	24	2.68	1.9
AISI 1050	37	39	23	2.54	1.84

Table 5. Results of MSS method

	Solid	Hollow		Solid	Hollow
AISI / CD	d (mm)	do (mm)	di (mm)	mass (kg)	
AISI 1010	43	45	27	3.43	2.4
AISI 1018	40	42	25	2.97	2.11
AISI 1020	40	41	24	2.97	2.05
AISI 1030	38	40	24	2.68	1.9
AISI 1035	37	39	23	2.54	1.84
AISI 1040	37	38	22	2.54	1.78
AISI 1050	35	36	21	2.27	1.58

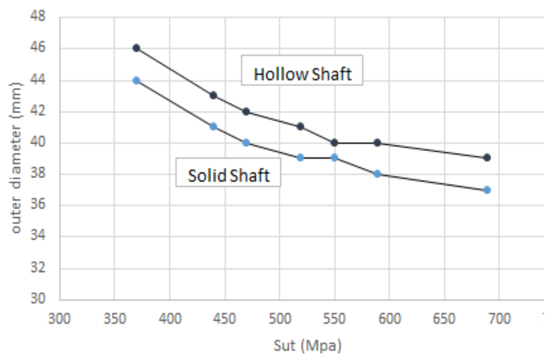

Figure 3: Designed diameter for solid and hollow shafts using Mod-Goodman method..

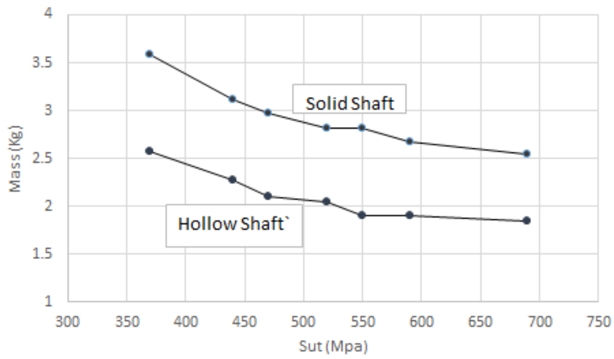

Figure 2: Mass of designed sloid and hollow shafts using Mod-Goodman method

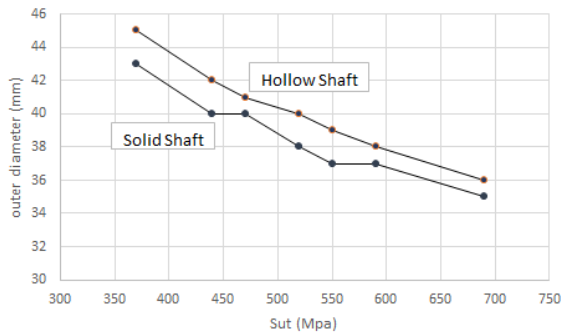

Figure 5: Designed diameter for solid and hollow shafts using MSS method

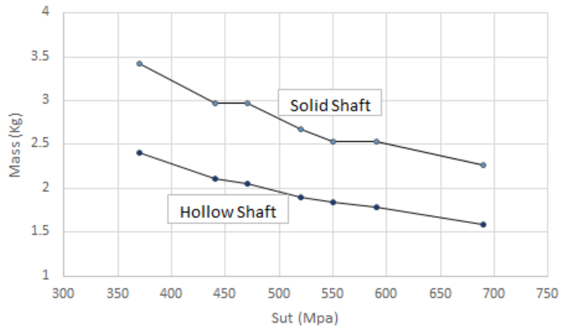

Figure 6: Mass of designed sloid and hollow shafts using MSS method

Table 6 depicts the Von Mises component results (σ_a', σ_m') , which are represented in Figures 7 and 8. They show that solid and hollow shafts in this study have nearly identical Von Mises stress components, and the factor of safety against first-cycle yield is bigger than the fatigue factor of safety (n=2.5). As a result, there is no localized yielding, and fatigue will come first. The Von Mises component results were plotted on Mod-Goodman lines, as illustrated in Figures 9 and 10. According to those figures, the working points in our case are below the Yield (langer) and Mod-Goodman lines, indicating that the design is free of first-cycle yielding and has infinite life.

Table 6. Results of Von Mises components calculation

	Solid			Hollow		
AISI / CD	σ_a' MPa	σ_m' MPa	n_y	σ_a' MPa	σ_m' MPa	n_y
AISI 1010	19.2	93.25	2.67	19.31	93.75	2.65
AISI 1018	23.73	115.25	2.66	23.64	114.78	2.67
AISI 1020	25.56	124.11	2.61	25.36	123.18	2.68
AISI 1030	27.57	133.91	2.72	27.27	132.41	2.76
AISI 1035	27.57	133.91	2.85	29.36	142.59	2.68
AISI 1040	29.81	144.76	2.81	29.36	142.59	2.85
AISI 1050	32.29	156.82	3.07	31.68	153.84	3.13

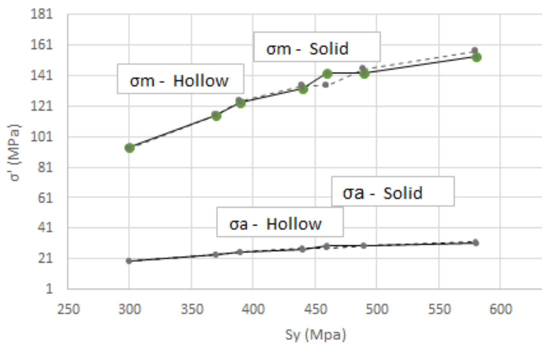

Figure 7: Von Mises stress components

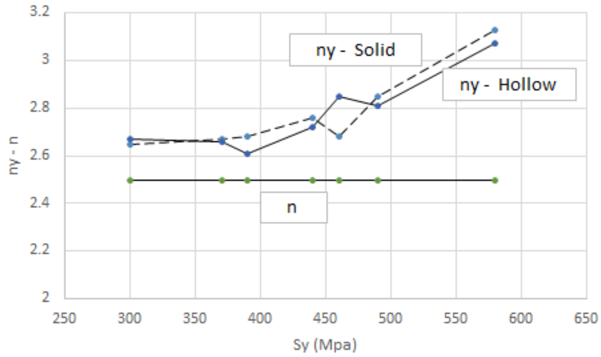

Figure 8: factor of safety guarding against first-cycle yield

VII. CONCLUSION

The results show a considerable reduction in the mass of hollow shafts compared to solid shafts with the same material and strength of 28% on average, while the outer diameter of hollow shafts increases by roughly 5% when compared to solid shafts. The results show a minor increase in the diameter of the proposed shaft when maximum share stress method is employed compared to Mod-Goodman method since the fatigue factor of safety is not applied in the first method. Furthermore, calculations show that the design is safe and has infinite Fatigue life for both solid and hollow shafts, with all working points falling below Mod-Goodman and Langer lines.

VIII. RECOMMENDATIONS

An experimental investigation should be conducted to support the results of this study, and software such as ANSYS should be implemented.

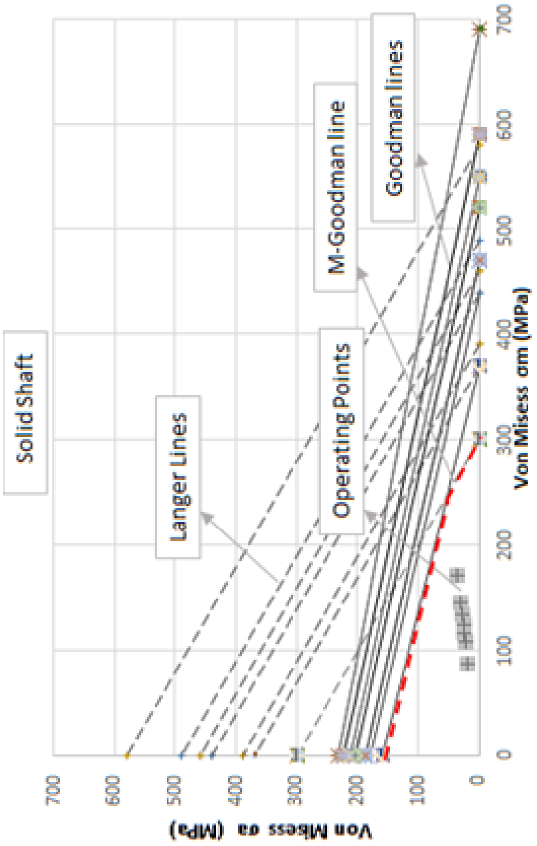

Figure 9: Fatigue diagram shows Mod-Goodman lines (Solid Shafts)

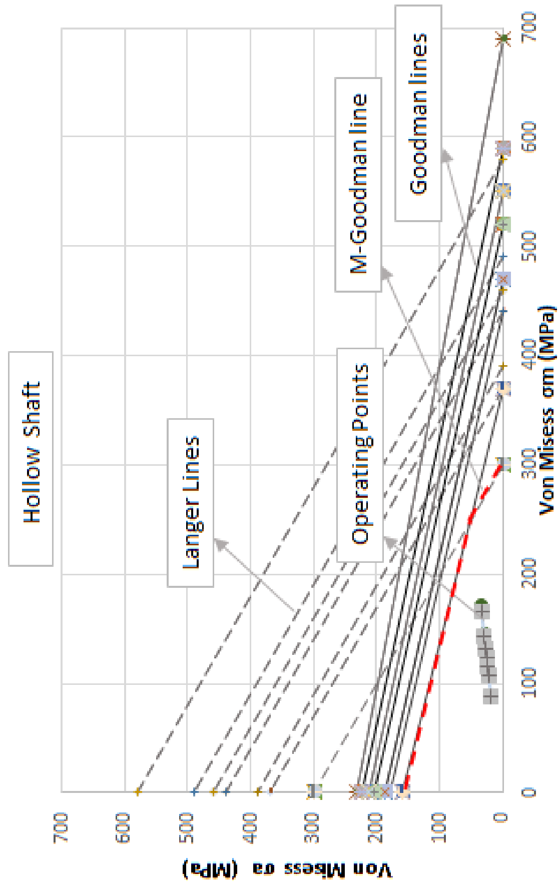

Figure 10 Fatigue diagram shows Mod-Goodman lines (Hollow Shafts)

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